Adding and Subtracting Integers

In Investigation 1, you used number lines and chip boards to model operations with integers. Now, you will develop algorithms for adding and subtracting integers.

An **algorithm** is a plan, or series of steps, for doing a computation. In an effective algorithm, the steps lead to the correct answer, no matter what numbers you use. You may even develop more than one algorithm for each computation. Your goal should be to understand and skillfully use at least one algorithm for adding integers and at least one algorithm for subtracting integers.
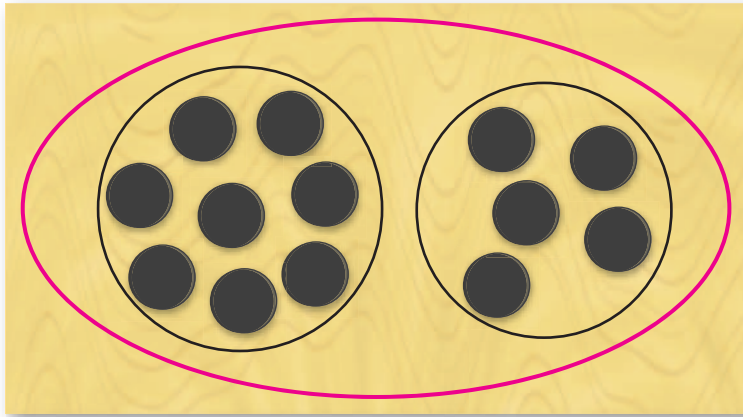
2.1 Introducing Addition of Integers

There are two common ways that number problems lead to addition calculations like $8 + 5$. The first involves combining two similar sets of objects, like in this example:

> John has 8 video games and his friend has 5. Together they have $8 + 5 = 13$ games.

You can represent this situation on a chip board.

8 + 5 = 13

Number problems also lead to addition calculations when you add to a starting number. Take the following example:

At a desert weather station, the temperature at sunrise was 10°C. It rose 25°C by noon. The temperature at noon was 10°C + 25°C = 35°C.

You can represent this situation on a number line. The starting point is $^+10$. The change in distance and direction is $^+25$. The sum ($^+35$) is the result of moving that distance and direction.
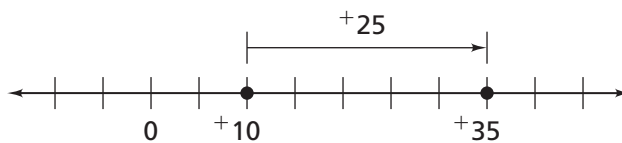
Suppose, instead of rising 25°C, the temperature fell 15°C. The next number line shows that $^+10$°C + $^-15$°C = $^-5$°C.
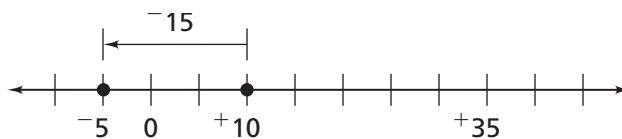
Use these ideas about addition as you develop an algorithm for addition of integers.

Use chip models or number line models.

A. **1.** Find the sums in each group.

    2. Describe what the examples in each group have in common.

    3. Use your answer to part (2) to write two problems for each group.

    4. Describe an algorithm for adding integers in each group.

Group 1	Group 2
$^+2 + {}^+8$	$^+8 + {}^-12$
$^-3 + {}^-8$	$^-3 + {}^+2$
$^+20 + {}^+25$	$^+14 + {}^-23$
$^-24 + {}^-12$	$^-11 + {}^+13$

B. Write each number as a sum of integers in three different ways.

    1. $^-5$ **2.** $^+15$ **3.** 0

    4. Check to see whether your strategy for addition of integers works on these rational number problems.

        a. $^-1 + {}^+9$ **b.** $^-1\frac{1}{2} + {}^-\frac{3}{4}$ **c.** $^+1\frac{1}{2} + {}^-2\frac{3}{4}$

C. Write a story to match each number sentence. Find the solutions.

    1. $^+50 + {}^-65 = \blacksquare$ **2.** $^-15 + \blacksquare = {}^-25$ **3.** $^-300 + {}^-250 = \blacksquare$

D. Find both sums in parts (1) and (2). What do you notice?

    1. $^+12 + {}^-35$ $^-35 + {}^+12$ **2.** $^-7\frac{2}{3} + {}^-1\frac{1}{6}$ $^-1\frac{1}{6} + {}^-7\frac{2}{3}$

    3. The property of rational numbers that you have observed is called the **Commutative Property** of addition. What do you think the Commutative Property says about addition of rational numbers?

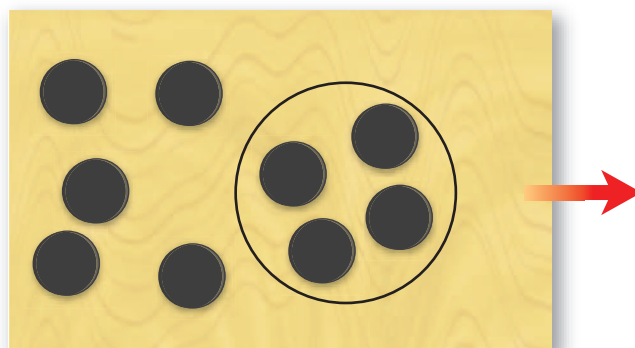**ACE** **Homework starts on page 32.**

2.2 Introducing Subtraction of Integers

In some subtraction problems, you *take away* objects from a set, as in this first example:

    Example 1 Kim had 9 CDs. She sold 4 CDs at a yard sale. She now has only $9 - 4 = 5$ of those CDs left.
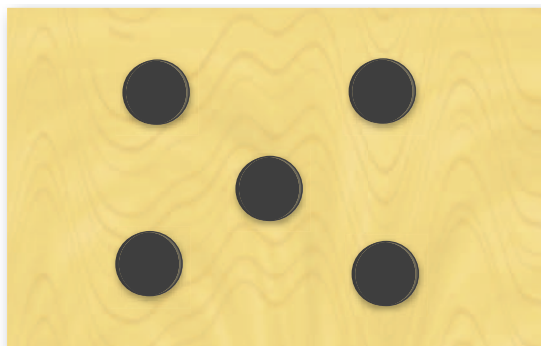
9 – 4 = 5

You can represent this situation on a chip board.

Here is another example.

Example 2 Otis earned $5 babysitting. He owes Latoya $7. He pays her the $5. Represent this integer subtraction on a chip board.

To subtract 7 from 5 ($^{+}5 - {}^{+}7$), start by showing $^{+}5$ as black chips.

You can't take away $^{+}7$ because there aren't seven black chips to remove. Since adding both a red chip and a black chip does not change the value of the board, add two black chips and two red chips. The value of the board stays the same, but now there are 7 black chips to take away.
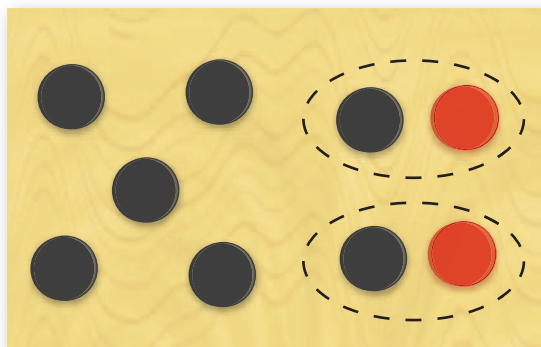
5 – 7 = $^{-}$2

What is left on the board when you take away the 7 black chips?

The changes on the board can be represented by $(^{-}2 + 2) + 5 - 7 = {}^{-}2$. Otis now has $^{-}$$2. He still owes Latoya $2.

In a third example of a subtraction problem, you find the *difference* between two numbers.

Example 3 The Arroyo family just passed mile 25 on the highway. They need to get to the exit at mile 80. How many more miles do they have to drive?
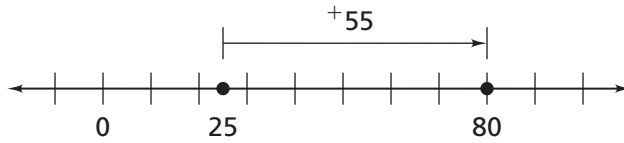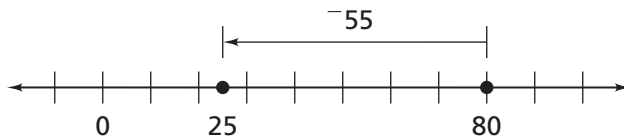
You can use a number line to show differences.

$$+55$$

0   25   80

The arrow on the number line points in the direction of travel. The Arroyos are traveling in a positive direction from small values to greater values. They still have to travel $80 - 25 = 55$ miles.

If the Arroyos drive back from mile 80 to mile 25, they still have to travel 55 miles. This time, however, they travel in the opposite direction. The number sentence $25 - 80 = {}^-55$ represents this situation.

$$^-55$$

0   25   80

Now, the arrow points to the left and has a label of $^-55$. The distance is 55, but the direction is negative.

Sometimes you only want the distance and not direction. You can show distance by putting vertical bars around the given number. This is called absolute value. The **absolute value** of a number is its distance from 0 on the number line.

$$|{}^-55| = 55 \qquad\qquad |{}^+55| = 55$$

You say "the absolute value of $^-55$ is 55" and "the absolute value of $^+55$ is 55."

When you write a number and a sign (or an implied sign for $+$) on an arrow above a number line, you are indicating both distance and direction.

In a problem that involves the amount of money you have and the amount that you owe, is the sign (direction) important?

Problem 2.2 Introducing Subtraction of Integers

Use chip models or number line models.

A. 1. Find the differences in each group below.

Group 1
$^+12 - {}^+8$
$^-5 - {}^-7$
$^-4 - {}^-2$
$^+2 - {}^+4$

Group 2
$^+12 - {}^-8$
$^-5 - {}^+7$
$^-4 - {}^+2$
$^+2 - {}^-4$

   2. Describe what the examples in each group have in common.

   3. Use your answer to part (2) to write two problems for each group.

   4. Describe an algorithm for subtracting integers in each group.

   5. Check to see whether your strategy for subtraction of integers works on these rational number problems:

     a. $^-1 - {}^+3$ **b.** $^-1 - {}^+\frac{3}{4}$

     c. $^-1\frac{1}{2} - {}^-2$ **d.** $^-1\frac{1}{2} - {}^-\frac{3}{4}$

B. Write each number as a difference of integers in three different ways.

   1. $^-5$ **2.** $^+15$

   3. 0 **4.** $^-3.5$

C. For parts (1)–(4), decide whether the expressions are equal.

   1. $^-2 - {}^+3 \stackrel{?}{=} {}^+3 - {}^-2$ **2.** $^+12 - {}^-4 \stackrel{?}{=} {}^-4 - {}^+12$

   3. $^-15 - {}^-20 \stackrel{?}{=} {}^-20 - {}^-15$ **4.** $^+45 - {}^+21 \stackrel{?}{=} {}^+21 - {}^+45$

   5. Do you think there is a Commutative Property of subtraction?

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You have probably noticed that addition and subtraction are related to each other. You can write any addition sentence as an equivalent subtraction sentence. You can also write any subtraction sentence as an equivalent addition sentence.

Getting Ready for Problem 2.3

The chip board below shows a value of $^+5$.

- There are two possible moves, one addition and one subtraction, that would change the value on the board to $^+2$ in one step. How would you complete the number sentences to represent each move?

$$^+5 + \blacksquare = {}^+2 \text{ and } {}^+5 - \blacksquare = {}^+2$$

- There are two possible moves, one addition and one subtraction, that would change the value on the board to $^+8$ in one step. How would you complete the number sentences to represent each move?

$$^+5 + \blacksquare = {}^+8 \text{ and } {}^+5 - \blacksquare = {}^+8$$

- Can you describe a general relationship between addition and subtraction for integers?

Use your ideas about addition and subtraction of integers to explore the relationship between these two operations.

A. Complete each number sentence.

1. $^+5 + {}^-2 = {}^+5 - \blacksquare$

2. $^+5 + {}^+4 = {}^+5 - \blacksquare$

3. $^-7 + {}^-2 = {}^-7 - \blacksquare$

4. $^-7 + {}^+2 = {}^-7 - \blacksquare$

B. What patterns do you see in the results of Question A that suggest a way to restate any addition problem as an equivalent subtraction problem?

C. Complete each number sentence.

1. $^+8 - {}^+5 = 8 + \blacksquare$

2. $^+8 - {}^-5 = 8 + \blacksquare$

3. $^-4 - {}^+6 = {}^-4 + \blacksquare$

4. $^-4 - {}^-6 = {}^-4 + \blacksquare$

D. What patterns do you see in the results of Question C that suggest a way to restate any subtraction problem as an equivalent addition problem?

E. Write an equivalent problem for each. Then find the results.

1. $^+396 - {}^-400$

2. $^-75.8 - {}^-35.2$

3. $^-25.6 + {}^-4.4$

4. $^+\frac{3}{2} - {}^+\frac{1}{4}$

5. $^+\frac{5}{8} + {}^-\frac{3}{4}$

6. $^-3\frac{1}{2} - {}^+5$

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2.4 Fact Families

You can rewrite $3 + 2 = 5$ to make a fact family that shows how the addition sentence is related to two subtraction sentences.

$$3 + 2 = 5$$
$$2 + 3 = 5$$
$$5 - 3 = 2$$
$$5 - 2 = 3$$

Problem 2.4 Fact Families

A. Write a related subtraction fact for each.

    1. $^-3 + {}^-2 = {}^-5$ **2.** $^+25 + {}^-32 = {}^-7$

B. Write a related addition fact for each.

    1. $^+8 - {}^-2 = {}^+10$ **2.** $^-14 - {}^-20 = 6$

C. 1. Write a related sentence for each.

    a. $n - {}^+5 = {}^+35$ **b.** $n - {}^-5 = {}^+35$ **c.** $n + {}^+5 = {}^+35$

    2. Do your related sentences make it easier to find the value for n? Why or why not?

D. 1. Write a related sentence for each.

    a. $^+4 + n = {}^+43$ **b.** $^-4 + n = {}^+43$ **c.** $^-4 + n = {}^-43$
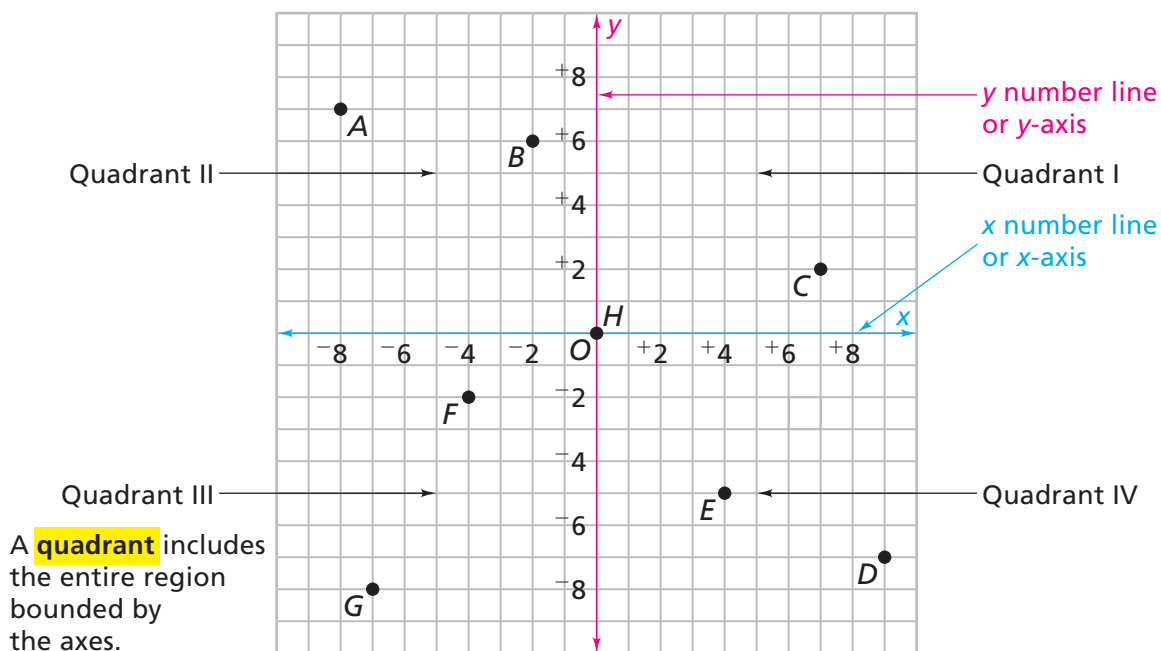
    2. Do your related sentences make it easier to find the value for n? Why or why not?

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2.5 Coordinate Graphing

In your study of similar figures, you used positive number coordinates and arithmetic operations to locate and move points and figures around a coordinate grid. You can use negative number coordinates to produce a grid that extends in all directions.

Coordinate Plane

- **y number line or y-axis**
- **Quadrant II**
- **Quadrant I**
- **x number line or x-axis**
- **Quadrant III**
- **Quadrant IV**

A **quadrant** includes the entire region bounded by the axes.

Problem 2.5 Coordinate Graphing

A. Write the coordinates for each point labeled with a letter.

B. What is the sign of the *x*-value and the *y*-value for any point in Quadrant I? Quadrant II? Quadrant III? Quadrant IV?

C. The point "opposite" ($^-5$, $^+8$) has coordinates ($^+5$, $^-8$). Notice that the sign of each coordinate in the pair changes. Write the coordinates for the points "opposite" the labeled points. On a grid like the one shown, graph and label each "opposite" point with a letter followed by a tick mark. Point A′ is "opposite" point A.

D. Draw line segments connecting each pair of related points (A and A′, B and B′, etc.). What do you notice about the line segments?

E. Plot the points in each part on a grid. Connect the points to form a triangle. Draw each triangle in a different color, but on the same grid.

  1. ($^+1$, $^-1$)   ($^+2$, $^+3$)   ($^-4$, $^-2$)

  2. ($^-1$, $^-1$)   ($^-2$, $^+3$)   ($^+4$, $^-2$)

  3. ($^-1$, $^+1$)   ($^-2$, $^-3$)   ($^+4$, $^+2$)

  4. ($^+1$, $^+1$)   (2, $^-3$)   ($^-4$, $^+2$)

  5. How is triangle 1 related to triangle 2? How is triangle 1 related to triangle 3? To triangle 4?

ACE **Homework starts on page 32.**